ANSWERS

LESSON 1.1 • Building Blocks of Geometry

1. S  2. 9 cm  3. SN  4. endpoint
5. NS  6. PQ  7. SP
8. KN \equiv KL, NM \equiv LM, NO \equiv LO
9. E(−14, 15)
10.

11.

12. \( \overline{AB}, \overline{AC}, \overline{AD}, \overline{AE}, \overline{AF}, \overline{BC}, \overline{BD}, \overline{BE}, \overline{BF}, \overline{CD}, \overline{CE}, \overline{CF}, \overline{DE}, \overline{DF}, \overline{EF} \) (15 lines)

13. Possible coplanar set: \{C, D, H, G\}; 12 different sets

LESSON 1.2 • Poolroom Math

1. vertex  2. bisector  3. side
4. 126°  5. \( \angle DAE \)  6. 133°
7. 47°  8. 63°  9. 70°
10.

11.

12.

LESSON 1.3 • What’s a Widget?

1. d  2. c  3. e  4. i
5. f  6. b  7. h  8. a
9. g
10. They have the same measure, 13°. Because \( m\angle Q = 77° \), its complement has measure 13°. So \( m\angle R = 13° \), which is the same as \( m\angle P \).

11.

12.

LESSON 1.4 • Polygons

<table>
<thead>
<tr>
<th>Polygon name</th>
<th>Number of sides</th>
<th>Number of diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Triangle</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2. Quadrilateral</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3. Pentagon</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>4. Hexagon</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>5. Heptagon</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>6. Octagon</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>7. Decagon</td>
<td>10</td>
<td>35</td>
</tr>
<tr>
<td>8. Dodecagon</td>
<td>12</td>
<td>54</td>
</tr>
</tbody>
</table>

13.

14. 120°  15. 75°
11. $\overline{AC}, \overline{AD}, \overline{AE}$

12. Possible answer: $\overline{AB}$ and $\overline{BC}$

13. Possible answer: $\angle A$ and $\angle B$

14. Possible answer: $\overline{AC}$ and $\overline{FD}$

15. 82°  16. 7.2  17. 61°  18. 16.1

19. 6.2 cm

**LESSON 1.5 • Triangles**

For Exercises 1–7, answers will vary. Possible answers are shown.

1. $\overline{AB} \parallel \overline{GH}$

2. $\overline{EF} \perp \overline{BI}$

3. $\overline{CG} \equiv \overline{FH}$

4. $\angle DEG$ and $\angle GEF$

5. $\angle DEG$ and $\angle GEF$

6. 

7. 

8. 

9. 

For Exercises 10–12, answers may vary. Possible answers are shown.

10. $F(8, -2)$

11. $D(4, 3)$

12. $G(10, -2)$

**LESSON 1.6 • Special Quadrilaterals**

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. The chord goes through the center, $P$. (It is a diameter.)

10. 

For Exercises 6–10, 12, and 13, answers may vary. Possible answers are shown.

6. 

7. $ACFD$  8. $EFHG$  9. $BFJD$  10. $BFHD$

11. $D(0, 3)$  12. $E(0, 5)$  13. $G(16, 3)$

**LESSON 1.7 • Circles**

1. 48°  2. 132°  3. 228°  4. 312°

5. 

6. 

7. $(8, 2); (3, 7); (3, -3)$

8. 

9. 

10. 

ANSWERS
LESSON 1.8 • Space Geometry

1. Rectangular prism

2. Pentagonal prism

3. Rectangular prism

4. Pentagonal prism

5. 18 cubes

6. $x = 2, \ y = 1$

LESSON 1.9 • A Picture Is Worth a Thousand Words

1. Possible locations

2. Dora, Ellen, Charles, Anica, Fred, Bruce

3. Triangles

   - Acute triangles
   - Isosceles triangles
   - Scalene triangles

LESSON 2.1 • Inductive Reasoning

4. Possible answers:
   a. 
   b. 
   c. 

   

   

   

5. 20, 24

6. $12\frac{1}{2}, 6\frac{1}{4}$

7. $\frac{5}{4}, 2$

8. $-1, -1$

9. 72, 60

10. 243, 729

11. 91, 140

12. False; $11 \cdot 10 = 110, 11 \cdot 12 = 132$

13. True

LESSON 2.2 • Finding the $n^{th}$ Term

1. Linear

2. Linear

3. Not linear

4. Linear

5. $n \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$
   
   $f(n) \quad -5 \quad 2 \quad 9 \quad 16 \quad 23$

6. $n \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$
   
   $g(n) \quad -10 \quad -18 \quad -26 \quad -34 \quad -42$

7. $f(n) = 4n + 5; f(50) = 205$

8. $f(n) = -5n + 11; f(50) = -239$

9. $f(n) = \frac{1}{2}n + 6; f(50) = 31$
4. Answers will vary. Possible answers:

![Diagram of triangles and reasoning]

LESSON 2.4 • Deductive Reasoning

1. No. Explanations will vary. Sample explanation:
   Because \( \triangle ABC \) is equilateral, \( AB = BC \). Because \( C \) lies between \( B \) and \( D \), \( BD > BC \), so \( BD \) is not equal to \( AB \). Thus \( \triangle ABD \) is not equilateral, by deductive reasoning.

2. Answers will vary. \( m \angle E > m \angle D \) (\( m \angle E = m \angle D + 90^\circ \)); deductive

3. a, e, f; inductive

4. Deductive
   a. \( 4x + 3(2 - x) = 8 - 2x \) The original equation.
      \[ 4x + 6 - 3x = 8 - 2x \] Distributive property.
      \[ x + 6 = 8 - 2x \] Combining like terms.
      \[ 3x + 6 = 8 \] Addition property of equality.
      \[ 3x = 2 \] Subtraction property of equality.
      \[ x = \frac{2}{3} \] Division property of equality.

LESSON 2.3 • Mathematical Modeling

1. 

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>\ldots</th>
<th>( n )</th>
<th>\ldots</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of triangles</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
<td>\ldots</td>
<td>( 4n - 3 )</td>
<td>\ldots</td>
<td>197</td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>\ldots</th>
<th>( n )</th>
<th>\ldots</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of figure</td>
<td>1</td>
<td>4</td>
<td>16</td>
<td>64</td>
<td>256</td>
<td>\ldots</td>
<td>( 4^{n-1} )</td>
<td>\ldots</td>
<td>4^{49}</td>
</tr>
</tbody>
</table>

10.  

![Grid diagram]

a. 240
b. 1350
c. \( f(n) = 2n(n + 2) \), or \( f(n) = 2n^2 + 4n \)

11.  

![Diagram of sequences]

a. 8 sequences
b. 3 sequences have 1 tail.
c. \( \frac{3}{8} \)

3. 66 different pairs. Use a dodecagon showing sides and diagonals.
b. \( \frac{19 - 2(3x - 1)}{5} = x + 2 \)  
The original equation.

\( 19 - 2(3x - 1) = 5(x + 2) \)  
Multiplication property of equality.

\( 19 - 6x + 2 = 5x + 10 \)  
Distributive property.

\( 21 - 6x = 5x + 10 \)  
Combining like terms.

\( 21 = 11x + 10 \)  
Addition property of equality.

\( 11 = 11x \)  
Subtraction property of equality.

\( 1 = x \)  
Division property of equality.

5. a. 16, 21; inductive
   b. \( f(n) = 5n - 9; 241; \) deductive

**LESSON 2.5 • Angle Relationships**

1. \( a = 68^\circ, b = 112^\circ, c = 68^\circ \)
2. \( a = 127^\circ \)
3. \( a = 35^\circ, b = 40^\circ, c = 35^\circ, d = 70^\circ \)
4. \( a = 90^\circ, b = 90^\circ, c = 42^\circ, d = 48^\circ, e = 132^\circ \)
5. \( a = 20^\circ, b = 70^\circ, c = 20^\circ, d = 70^\circ, e = 110^\circ \)
6. \( a = 70^\circ, b = 55^\circ, c = 25^\circ \)
7. Sometimes  
8. Always  
9. Never  
10. Sometimes  
11. acute  
12. 158°  
13. 90°  
14. obtuse  
15. converse

**LESSON 2.6 • Special Angles on Parallel Lines**

1. \( a = 54^\circ, b = 54^\circ, c = 54^\circ \)
2. \( a = 115^\circ, b = 65^\circ, c = 115^\circ, d = 65^\circ \)
3. \( a = 72^\circ, b = 126^\circ \)
4. \( \ell_1 \parallel \ell_2 \)
5. \( \ell_1 \parallel \ell_2 \)
6. cannot be determined  
7. \( a = 102^\circ, b = 78^\circ, c = 58^\circ, d = 122^\circ, e = 26^\circ, f = 58^\circ \)
8. \( x = 80^\circ \)
9. \( x = 20^\circ, y = 25^\circ \)
LESSON 3.3 • Constructing Perpendiculars to a Line

1. False. The altitude from \( A \) coincides with the side so it is not shorter.

2. False. In an isosceles triangle, an altitude and median coincide so they are of equal length.

3. True

4. False. In an acute triangle, all altitudes are inside. In a right triangle, one altitude is inside and two are sides. In an obtuse triangle, one altitude is inside and two are outside. There is no other possibility so exactly one altitude is never outside.

5. False. In an obtuse triangle, the intersection of the perpendicular bisectors is outside the triangle.

6. \( \triangle ABC \) is not unique.

7. \( \triangle ABC \) is not unique.

8. \( BD = AD = CD \)

9. \( WX = YZ \)

7. a. \( A \) and \( B \)
   b. \( A, B, \) and \( C \)
   c. \( A \) and \( B \) and from \( C \) and \( D \) (but not from \( B \) and \( C \))
   d. \( A \) and \( B \) and from \( D \) and \( E \)
LESSON 3.4 • Constructing Angle Bisectors

1. a. \( \ell_1 \) and \( \ell_2 \)
   b. \( \ell_1, \ell_2, \) and \( \ell_3 \)
   c. \( \ell_2, \ell_3, \) and \( \ell_4 \)
   d. \( \ell_1 \) and \( \ell_2 \) and from \( \ell_3 \) and \( \ell_4 \)

2. \( \overline{AP} \) is the bisector of \( \angle CAB \)

3. \( x = 20^\circ, m\angle ABE = 50^\circ \)

4. 

5. They are concurrent.

6. 

7. \( RN = GN \) and \( RO = HO \)

LESSON 3.5 • Constructing Parallel Lines

1. 

2. 

3. 

4. 

LESSON 3.6 • Construction Problems

1. Possible answer:

2. Possible answer:

5. 

6. Possible answer:
3. Possible answers:

4. Possible answer:

5. Possible answer:

6. Possible answer:

7. Possible answer:

8. Possible answer:

**LESSON 3.7 • Constructing Points of Concurrency**

1. Circumcenter

2. Locate the power-generation plant at the incenter. Locate each transformer at the foot of the perpendicular from the incenter to each side.

3. Possible answer: In the equilateral triangle, the centers of the inscribed and circumscribed circles are the same. In the obtuse triangle, one center is outside the triangle.

4. Possible answer: In the equilateral triangle, the centers of the inscribed and circumscribed circles are the same. In the obtuse triangle, one center is outside the triangle.

5. Possible answer: In an acute triangle, the circumcenter is inside the triangle. In a right triangle, it is on the hypotenuse. In an obtuse triangle, the circumcenter is outside the triangle. (Constructions not shown.)
LESSON 3.8 • The Centroid
1. 
2. 
3. $CP = 3.3 \text{ cm, } CQ = 5.7 \text{ cm, } CR = 4.8 \text{ cm}$

LESSON 4.2 • Properties of Isosceles Triangles
1. $m \angle T = 64^\circ$
2. $m \angle G = 45^\circ$
3. $x = 125^\circ$
4. $m \angle A = 39^\circ$, perimeter of $\triangle ABC = 46 \text{ cm}$
5. $LM = 163 \text{ m, } m \angle M = 50^\circ$
6. $m \angle Q = 44^\circ$, $QR = 125$
7. a. $\angle DAB \equiv \angle ABD \equiv \angle BDC \equiv \angle CBD$
   b. $\angle ADB \equiv \angle CBD$
   c. $AD \parallel BC$ by the Converse of the AIA Conjecture.
8. $x = 21^\circ$, $y = 16^\circ$
9. $m \angle QPR = 15^\circ$
10. $m \angle PRQ = 55^\circ$ by VA, which makes $m \angle P = 55^\circ$ by the Triangle Sum Conjecture. So, $\triangle PQR$ is isosceles by the Converse of the Isosceles Triangle Conjecture.

LESSON 4.3 • Triangle Inequalities
1. Yes
2. No
3. $19 < x < 53$
4. $b > a > c$
5. $b > c > a$
6. $a > c = d > b$
7. $x = 76^\circ$
8. $x = 79^\circ$
9. The interior angle at $A$ is $60^\circ$. The interior angle at $B$ is $20^\circ$. But now the sum of the measures of the triangle is not $180^\circ$.
10. By the Exterior Angles Conjecture, $2x = x + m \angle PQS$. So, $m \angle PQS = x$. So, by the Converse of the Isosceles Triangle Conjecture, $\triangle PQS$ is isosceles.
11. Not possible. $AB + BC < AC$
12. 

LESSON 4.4 • Are There Congruence Shortcuts?
1. SAA or ASA
2. SSS
3. SSS
4. $\triangle BQM$ (SAS)
5. $\triangle TIE$ (SSS)
9. All triangles will be congruent by ASA. Possible triangle:

10. All triangles will be congruent by SAA. Possible procedure: Use $\angle A$ and $\angle C$ to construct $\angle B$ and then copy $\angle A$ and $\angle B$ at the ends of $\overline{AB}$.

LESSON 4.6 • Corresponding Parts of Congruent Triangles

1. SSS, SAS, ASA, SAA
2. $YZ \parallel WX$, AIA Conjecture
3. $WZ \parallel XY$, AIA Conjecture
4. ASA
5. CPCTC
6. $\triangle YWM \equiv \triangle ZXM$ by SAS. $\overline{YW} \equiv \overline{ZX}$ by CPCTC.
7. $\triangle ACD \equiv \triangle BCD$ by SAS. $\overline{AD} \equiv \overline{BD}$ by CPCTC.
8. Possible answer: $DE$ and $CF$ are both the distance between $\overline{DC}$ and $\overline{AB}$. Because the lines are parallel, the distances are equal. So, $DE \equiv CF$.
9. Possible answer: $\overline{DE} \equiv \overline{CF}$ (see Exercise 8), $\angle DEF \equiv \angle CFE$ because both are right angles, $\overline{EF} \equiv \overline{FE}$ because they are the same segment. So, $\triangle DEF \equiv \triangle CFE$ by SAS. $\overline{EC} \equiv \overline{FD}$ by CPCTC.
10. Possible answer: It is given that $TP = RA$ and $\angle PTR \equiv \angle ART$, and $\overline{TR} \equiv \overline{RT}$ because they are the same segment. So $\triangle PTR \equiv \triangle ART$ by SAS and $\overline{TA} \equiv \overline{RP}$ by CPCTC.

LESSON 4.7 • Flowchart Thinking

1. (See flowchart proof at bottom of page 101.)
2. (See flowchart proof at bottom of page 101.)
3. (See flowchart proof at bottom of page 101.)

LESSON 4.8 • Proving Special Triangle Conjectures

1. $AD = 8$
2. $m\angle ACD = 36^\circ$
3. $m\angle B = 52^\circ$, $CB = 13$
4. $m\angle E = 60^\circ$
5. $AN = 17$
6. Perimeter $ABCD = 104$
7. (See flowchart proof at bottom of page 102.)

8. Flowchart Proof

\(\overline{CD}\) is a median

\[
\begin{align*}
\overline{AC} &= \overline{BC} & \text{Given} \\
\overline{AD} &= \overline{BD} & \text{Definition of median} \\
\overline{CD} &= \overline{CD} & \text{Same segment} \\
\triangle ADC &= \triangle BDC & \text{SSS Conjecture} \\
\angle ACD &= \angle BCD & \text{CPCTC} \\
\overline{CD} &= \text{bisects} & \angle ACB & \text{Definition of bisect}
\end{align*}
\]

LESSON 5.1 • Polygon Sum Conjecture

1. \(a = 103^\circ, b = 103^\circ, c = 97^\circ, d = 83^\circ, e = 154^\circ\)
2. \(a = 92^\circ, b = 44^\circ, c = 51^\circ, d = 85^\circ, e = 44^\circ, f = 136^\circ\)

Lesson 4.7, Exercises 1, 2, 3

1. \(PQ = SR\)

\[
\begin{align*}
PQ &\parallel SR & \text{Given} \\
PQ &= SR & \text{AIA Conjecture} \\
PQ &= QS & \text{Same segment} \\
\angle PQS &= \angle RSQ & \text{SSS Conjecture} \\
\triangle PQS &= \triangle RSQ & \text{CPCTC}
\end{align*}
\]

2. \(KTE\) is a kite

\[
\begin{align*}
KTE &= \text{kite} & \text{Given} \\
\angle KTE &= \angle KTI & \text{Definition of kite} \\
\angle KTE &= \angle KTI & \text{SSS Conjecture} \\
\angle KTE &= \angle KTI & \text{CPCTC} \\
KT &= \text{bisects} & \angle KET & \text{Definition of bisect}
\end{align*}
\]

3. \(ABCD\) is a parallelogram

\[
\begin{align*}
ABCD &= \text{parallelagram} & \text{Given} \\
\angle ABD &= \angle CDB & \text{AIA Conjecture} \\
\angle ABD &= \angle CDB & \text{Definition of parallelogram} \\
\angle ABD &= \angle CBD & \text{AIA Conjecture} \\
\angle ABD &= \angle CBD & \text{Definition of parallelogram} \\
\angle ABD &= \angle CBD & \text{AIA Conjecture} \\
\triangle BDA &= \triangle DBC & \text{ASA Conjecture} \\
\angle A &= \angle C & \text{CPCTC}
\end{align*}
\]
LESSON 5.3 • Kite and Trapezoid Properties

1. \( x = 30 \)  
2. \( x = 124^\circ, y = 56^\circ \)  
3. \( x = 64^\circ, y = 43^\circ \)  
4. \( x = 12^\circ, y = 49^\circ \)  
5. \( PS = 33 \)  
6. \( a > 11 \)

7. \( \text{AMNO is a parallelogram. By the Triangle Midsegment Conjecture, } \overline{ON} \parallel \overline{AM} \text{ and } \overline{MN} \parallel \overline{AO}. \)

Flowchart Proof

8. Paragraph proof: Looking at \( \triangle FGR, \overline{HI} \parallel \overline{FG} \) by the Triangle Midsegment Conjecture. Looking at \( \triangle PQR, \overline{FG} \parallel \overline{PQ} \) for the same reason. Because \( \overline{FG} \parallel \overline{PQ} \), quadrilateral \( FGQP \) is a trapezoid and \( DE \) is the midsegment, so it is parallel to \( FG \) and \( PQ \). Therefore, \( \overline{HI} \parallel \overline{FG} \parallel \overline{DE} \parallel \overline{PQ} \).

LESSON 5.4 • Properties of Midsegments

1. \( a = 89^\circ, b = 54^\circ, c = 91^\circ \)  
2. \( x = 21, y = 7, z = 32 \)  
3. \( x = 17, y = 11, z = 6.5 \)  
4. Perimeter \( \triangle XYZ = 66, PQ = 37, ZX = 27.5 \)  
5. \( M(12, 6), N(14.5, 2), \overline{AB} = -1.6, \overline{MN} = 1.6 \)  
6. Pick a point \( P \) from which \( A \) and \( B \) can be viewed over land. Measure \( AP \) and \( BP \) and find the midpoints \( M \) and \( N \). \( AB = 2MN \).

Lesson 4.8, Exercise 7

\( \overline{CD} \) is an altitude

\( \overline{AC} = \overline{BC} \)

\( \triangle ADC \) and \( \triangle BDC \) are right angles

\( \triangle ADC \cong \triangle BDC \)

\( \angle ADC \cong \angle BDC \)

\( \angle A \cong \angle B \)

\( \angle ADR \cong \angle BRD \)

\( \overline{AD} = \overline{BD} \)

\( \overline{CD} \) is a median

\( \overline{CD} = \overline{CD} \)

\( \overline{CD} \) is an altitude

\( \angle ADC \) and \( \angle BDC \) are right angles

\( \overline{AC} = \overline{BC} \)

\( \angle A \cong \angle B \)

\( \triangle ADC \cong \triangle BDC \)

\( \overline{ADC} = \overline{BDC} \)

\( \angle ADR = \angle BRD \)

\( \overline{AD} = \overline{BD} \)

\( \angle A \cong \angle B \)

\( \overline{CD} \) is a median

\( \overline{CD} \) is an altitude

\( \triangle ADC \cong \triangle BDC \)

\( \angle ADR = \angle BRD \)

\( \overline{AD} = \overline{BD} \)

\( \triangle ADC \cong \triangle BDC \)

\( \angle ADR = \angle BRD \)

\( \overline{AD} = \overline{BD} \)

\( \angle A \cong \angle B \)

\( \overline{CD} \) is a median
LESSON 5.5 • Properties of Parallelograms

1. Perimeter \(ABCD = 82\) cm
2. \(AC = 22, BD = 14\)
3. \(AB = 16, BC = 7\)
4. \(a = 51^\circ, b = 48^\circ, c = 70^\circ\)
5. \(AB = 35.5\)
6. \(a = 41^\circ, b = 86^\circ, c = 53^\circ\)
7. \(AD = 75\)
8. \(AD = 35.5\)
9. \(a = 38^\circ, b = 142^\circ, c = 142^\circ, d = 38^\circ, e = 142^\circ, f = 38^\circ, g = 52^\circ, h = 12^\circ, i = 61^\circ, j = 81^\circ, k = 61^\circ\)
10. No

LESSON 5.6 • Properties of Special Parallelograms

1. \(OQ = 16, m\angle QRS = 90^\circ, PR = 32\)
2. \(m\angle OKL = 45^\circ, m\angle MOL = 90^\circ,\) perimeter \(KLMN = 32\)
3. \(OB = 6, BC = 11, m\angle AOD = 90^\circ\)

Lesson 5.7, Exercise 1

LESSON 5.7 • Proving Quadrilateral Properties

1. (See flowchart proof at bottom of page.)
2. Flowchart Proof
   \[\begin{align*}
   &AB = CD \\
   &\text{Given} \\
   &AD = CD \\
   &\text{SSS Conjecture} \\
   &\iff \triangle ABD \cong \triangle CBD \\
   &\iff \angle A = \angle C \\
   &\text{CPCTC}
   \end{align*}\]
3. Flowchart Proof
   \[\begin{align*}
   &\triangle ABO = \triangle CBO = \triangle CDO = \triangle ADO \\
   &\text{SSS Conjecture} \\
   &\text{Diagonals of parallelogram bisect each other}
   \end{align*}\]
4. Flowchart Proof

- **AD \parallel BC**
  - Definition of parallelogram
- **\angle DAX = \angle BCY**
  - AIA Conjecture
- **AD = CB**
  - Opposite sides of parallelogram
- **\angle AXD = \angle CYB**
  - Both are 90°
- **\triangle AXD = \triangle CYB**
  - SAA Conjecture
- **DX = BY**
  - CPCTC

**LESSON 6.1 • Tangent Properties**

1. \( w = 126° \)
2. \( m \angle BQX = 65° \)
3. a. \( m \angle NQP = 90°, m \angle MPQ = 90° \)
   b. Trapezoid. Possible explanation: \( MP \) and \( NQ \) are both perpendicular to \( PQ \), so they are parallel to each other. The distance from \( M \) to \( PQ \) is \( MP \) and the distance from \( N \) to \( PQ \) is \( NQ \). But the two circles are not congruent, so \( MP \neq NQ \). Therefore, \( MN \) is not a constant distance from \( PQ \) and they are not parallel. Exactly one pair of sides is parallel, so \( MNQP \) is a trapezoid.
4. \( y = -\frac{1}{3}x + 10 \)
5. Possible answer: Tangent segments from a point to a circle are congruent. So, \( PA \equiv PB, PB \equiv PC \), and \( PC \equiv PD \). Therefore, \( PA \equiv PD \).
6. a. 4.85 cm
   b. 11.55 cm
7. **LES5ON 6.2 • Chord Properties**

- **a = 95°, b = 85°, c = 47.5°**
- **y** cannot be determined, \( w = 90° \)

**LESSON 6.3 • Arcs and Angles**

1. \( m \angle XNM = 40°, m \angle XN = 180°, mMN = 100° \)
2. \( x = 120°, y = 60°, z = 120° \)
3. \( a = 90°, b = 55°, c = 35° \)
4. \( a = 50°, b = 60°, c = 70° \)
5. \( x = 140° \)
6. \( m \angle A = 90°, m \angle AB = 72°, m \angle C = 36°, m \angle CB = 108° \)
7. \( m \angle AD = 140°, m \angle D = 30°, m \angle AB = 60°, m \angle DAB = 200° \)
8. \( p = 128°, q = 87°, r = 58°, s = 87° \)
9. \( a = 50°, b = 50°, c = 80°, d = 50°, e = 130°, f = 90°, g = 50°, h = 50°, j = 90°, k = 40°, m = 80°, n = 50° \)
4. Flowchart Proof
Construct radii \( \overline{AO}, \overline{OB}, \overline{OC}, \) and \( \overline{OD} \).

LESSON 6.5 • The Circumference/Diameter Ratio

1. \( C = 21\pi \) cm  
2. \( r = 12.5 \) cm  
3. \( C = 60\pi \) cm  
4. \( d = 24 \) cm  
5. \( C \approx 30.2 \) cm  
6. \( d \approx 42.0 \) cm, \( r \approx 21.0 \) cm  
7. \( C \approx 37.7 \) in.  
8. Yes; about 2.0 in.  
9. \( C \approx 75.4 \) cm  
10. Press the square against the tree as shown. Measure the tangent segment on the square. The tangent segment is the same length as the radius. Use \( C = 2\pi r \) to find the circumference.

11. 4 cm
LESSON 6.6 • Around the World

1. At least 7 olive pieces
2. About 2.5 rotations
3. \( \left( \frac{2\pi \cdot 4.23 \cdot 10^5}{60 \cdot 60 \cdot 23.93} \right) \approx 3085 \text{ m/s (about 3 km/s or just under 2 mi/s)} \)
4. 6.05 cm or 9.23 cm
5. Sitting speed = \( \frac{\left( \frac{2\pi \cdot 1.4957 \cdot 10^{15}}{10^3 \cdot 24} \right)}{364.25} \)
   \approx 107,500 \text{ km/h}

LESSON 6.7 • Arc Length

1. \( 4\pi \)
2. \( 4\pi \)
3. \( 30 \)
4. \( \frac{35\pi}{9} \)
5. \( \frac{80\pi}{9} \)
6. \( 6.25\pi \text{ or } \frac{25\pi}{4} \)
7. \( \frac{100\pi}{9} \)
8. \( 31.5 \)
9. \( 22\pi \)
10. \( 396 \)

EXPLORATION • Intersection Secants, Tangents, and Chords

1. \( x = 21^\circ \)
2. \( m\overarc{DC} = 70^\circ, m\overarc{ED} = 150^\circ \)
3. \( m\overarc{DC} = 114^\circ, m\angle DEC = 66^\circ \)
4. \( m\angle BCE = 75^\circ, m\overarc{BAC} = 210^\circ \)
5. \( x = 80^\circ, y = 110^\circ, z = 141^\circ \)
6. \( x = 34^\circ, y = 150^\circ, z = 122^\circ \)
7. \( x = 112^\circ, y = 68^\circ, z = 53^\circ \)
8. \( x = 28^\circ, y = 34.5^\circ \)

LESSON 7.1 • Transformations and Symmetry

1.
2.
LESSON 7.2 • Properties of Isometries

1. Rotation

2. Translation

3. Reflection

4. 

5. 

6. (x, y) → (x + 13, y + 6); translation; B'(8, 8), C'(-8, -4)

7. (x, y) → (-x, y); reflection across the y-axis; 
P'(7, -3), R(-4, 5)

8. (x, y) → (y, x); reflection across the line y = x; 
T'(7, 0), R(0, 3)

LESSON 7.3 • Compositions of Transformations

1. Translation by (-2, +5)

2. Rotation 45° counterclockwise

3. Translation by (+16, 0)

4. Rotation 180° about the intersection of the two lines

5. Translation by (-16, 0)

6. Rotation 180° about the intersection of the two lines

7. Reflection across the line x = -3

8. Reflection across the line x = 3

9. \( m\angle ROT = 50\°; \) rotation 100° clockwise about \( O \)

10. Translation 2 cm along the line perpendicular to \( k \) and \( \ell \) in the direction from \( k \) to \( \ell \).

11. 

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LESSON 7.4 • Tessellations with Regular Polygons

1. $n = 15$  
2. $n = 20$

3. Possible answer: A regular tessellation is a tessellation in which the tiles are congruent regular polygons whose edges exactly match.

4. Possible answer: A 1-uniform tiling is a tessellation in which all vertices are identical.

5. $3.4^2/6/3.6^3.6$

6. 

LESSONS 7.5–7.8 • Tessellations

1.

2. Sample answer:

3. Sample answer:

4. Sample answer:

LESSON 8.1 • Areas of Rectangles and Parallelograms

1. $112 \text{ cm}^2$  
2. $7.5 \text{ cm}^2$  
3. $110 \text{ cm}^2$  
4. $81 \text{ cm}^2$

5. $61 \text{ m}$

6. No. Possible answer:

7. $88 \text{ units}^2$  
8. $72 \text{ units}^2$

9. No. Carpet area is $20 \text{ yd}^2 = 180 \text{ ft}^2$. Room area is $(21.5 \text{ ft})(16.5 \text{ ft}) = 206.25 \text{ ft}^2$. Dana will be $26\frac{1}{2} \text{ ft}^2$ short.

LESSON 8.2 • Areas of Triangles, Trapezoids, and Kites

1. $16 \text{ ft}$  
2. $20 \text{ cm}^2$

3. $b = 12 \text{ in.}$  
4. $AD = 4.8 \text{ cm}$

5. $40 \text{ cm}^2$  
6. $88 \text{ cm}^2$

7. $54 \text{ units}^2$  
8. $135 \text{ cm}^2$

LESSON 8.3 • Area Problems

1. a. $549.5 \text{ ft}^2$  
   b. 40 bundles; $1596.00$

2. $500 \text{ L}$

3. Possible answer:
4. It is too late to change the area. The length of the diameters determines the area.

**LESSON 8.4 • Areas of Regular Polygons**

1. \(A \approx 696\ cm^2\)
2. \(a \approx 7.8\ cm\)
3. \(p \approx 43.6\ cm\)
4. \(n = 10\)
5. \(s = 4\ cm, a \approx 2.8\ cm, A \approx 28\ cm^2\)

**LESSON 8.5 • Areas of Circles**

1. \(81\pi\ cm^2\)
2. \(10.24\pi\ cm^2\)
3. \(23\ cm\)
4. \(324\pi\ cm^2\)
5. \(191.13\ cm^2\)
6. \(41.41\ cm^2\)
7. \(7.65\ cm^2\)
8. \(4.90\ cm^2\)
9. \(51.3\ cm^2\)
10. \(33.5\ or\ 33.6\ cm^2\)
11. \((64\pi - 128)\ square\ units\)
12. \(25\ pi\ cm^2\)

**LESSON 8.6 • Any Way You Slice It**

1. \(\frac{25\pi}{12}\ cm^2 \approx 6.54\ cm^2\)
2. \(\frac{52\pi}{3}\ cm^2 \approx 33.51\ cm^2\)
3. \(12\pi\ cm^2 \approx 37.70\ cm^2\)
4. \((16\pi - 32)\ cm^2 \approx 18.27\ cm^2\)
5. \(13.5\pi\ cm^2 \approx 42.41\ cm^2\)
6. \(10\pi\ cm^2 \approx 31.42\ cm^2\)
7. \(r = 10\ cm\)
8. \(x = 135^\circ\)
9. \(r = 7\ cm\)

**LESSON 8.7 • Surface Area**

1. \(136\ cm^2\)
2. \(240\ cm^2\)
3. \(558.1\ cm^2\)
4. \(796.4\ cm^2\)
5. \(255.6\ cm^2\)
6. \(356\ cm^2\)
7. \(468\ cm^2\)
8. \(1055.6\ cm^2\)
9. 1 sheet: front rectangle: \(3 \cdot \frac{1}{2} = 4\frac{1}{2}\); back rectangle: \(3 \cdot \frac{1}{2} = 6\); side trapezoids: \(2 \cdot \frac{1}{2} = 8\); total = 26 ft². Area of 1 sheet = \(4 \cdot 8 = 32\) ft². Possible pattern:

**LESSON 9.1 • The Theorem of Pythagoras**

1. \(a = 21\ cm\)
2. \(p \approx 23.9\ cm\)
3. \(x = 8\ ft\)
4. \(h \approx 14.3\ in.\)
5. \(Area \approx 19.0\ ft^2\)
6. \(C(11, -1); r = 5\)
7. \(Area \approx 49.7\ cm^2\)
8. \(RV \approx 15.4\ cm\)
9. If the base area is \(16\pi\ cm^2\), then the radius is 4 cm. The radius is a leg of the right triangle; the slant height is the hypotenuse. The leg cannot be longer than the hypotenuse.
10. \(Area = 150\ in^2\); hypotenuse \(QR = 25\ in.;\) altitude to the hypotenuse = 12 in.

**LESSON 9.2 • The Converse of the Pythagorean Theorem**

1. No
2. Yes
3. Yes
4. Yes
5. \(Area \approx 21.22\ cm^2\)
6. The top triangle is equilateral, so half its side length is 2.5. A triangle with sides 2.5, 6, and 6.5 is a right triangle because \(2.5^2 + 6^2 = 6.5^2\). So, the angle marked 95° should be 90°.
7. \(x \approx 44.45\ cm.\) By the Converse of the Pythagorean Theorem, \(\triangle ADC\) is a right triangle, and \(\angle ADC\) is a right angle. \(\angle ADC\) and \(\angle BDC\) are supplementary, so \(\angle BDC\) is also a right triangle. Use the Pythagorean Theorem to find \(x\).
8. 129.6 cm²

9. No. Because \( AB^2 + BC^2 \neq AC^2 \), \( \angle B \) of \( \triangle ABC \) is not a right angle.

10. Cannot be determined. The length of \( CD \) is unknown. One possible quadrilateral is shown.

11. Yes. Using SSS, \( \triangle ABC \cong \triangle BAD \cong \triangle CDA \cong \triangle DCB \). That means that the four angles of the quadrilateral are all congruent by CPCTC. Because the four angles must sum to 360° and they are all congruent, they must be right angles. So, \( ABCD \) is a rectangle.

**LESSON 9.3 • Two Special Right Triangles**

1. \( a = 14\sqrt{2} \) cm
2. \( a = 12 \) cm, \( b = 24 \) cm
3. \( a = 12 \) cm, \( b = 6\sqrt{3} \) cm
4. \( 64\sqrt{3} \) cm²
5. Perimeter = \( 32 + 6\sqrt{2} + 6\sqrt{3} \) cm; area = \( 60 + 18\sqrt{3} \) cm²
6. \( AC = 30\sqrt{2} \) cm; \( AB = 30 + 30\sqrt{3} \) cm; area = \( 450 + 450\sqrt{3} \) cm²
7. \( 45\sqrt{3} \) cm²
8. \( C\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\)
9. \( C(-6\sqrt{3}, -6)\)
10. Possible answer:

**LESSON 9.4 • Story Problems**

1. The foot is about 8.7 ft away from the base of the building. To lower it by 2 ft, move the foot an additional 3.3 ft away from the base of the building.

2. About 6.4 km

3. 149.5 linear feet of trim must be painted, or 224.3 feet². Two coats means 448.6 ft² of coverage. Just over 2 1/2 quarts of paint is needed. If Hans buys 3 quarts, he would have almost 1/2 quart left. It is slightly cheaper to buy 1 gallon and have about 1 1/2 quarts left. The choice is one of money versus conserving. Students may notice that the eaves extend beyond the exterior walls of the house and adjust their answer accordingly.

4. 14 in., \( \frac{14}{\sqrt{3}} \) in. ≈ 8.08 in., \( \frac{28}{\sqrt{3}} \) in. ≈ 16.17 in.

**LESSON 9.5 • Distance in Coordinate Geometry**

1. 10 units 2. 20 units 3. 17 units

4. \( ABCD \) is a rhombus: All sides = \( \sqrt{34} \), slope \( AB = \frac{-1}{2} \), slope \( BC = \frac{2}{3} \), so \( \angle B \) is not a right angle, and \( ABCD \) is not a square.

5. \( TUVW \) is an isosceles trapezoid: \( \overline{TU} \) and \( \overline{WV} \) have slope 1, so they are parallel. \( UV \) and \( TW \) have length \( \sqrt{20} \) and are not parallel (slope \( \overline{UV} = -\frac{1}{2} \), slope \( \overline{TW} = -2 \)).

6. Isosceles; perimeter = 32 units

7. \( M(7, 10); N(10, 14); \) slope \( \overline{MN} = \frac{4}{3} \), slope \( BC = \frac{4}{3} \), \( MN = 5 \), \( BC = 10 \); the slopes are equal; \( MN = \frac{1}{2} BC \).

8. \((x + 1)^2 + (y - 5)^2 = 4 \)

9. Center \((0, -2), r = 5\)

10. The distances from the center to the three points on the circle are not all the same: \( AP = \sqrt{61} \), \( BP = \sqrt{61} \), \( CP = \sqrt{52} \)
LESSON 9.6 • Circles and the Pythagorean Theorem
1. \(25\pi - 24\) cm\(^2\), or about 54.5 cm\(^2\)
2. \(72\sqrt{3} - 24\pi\) cm\(^3\), or about 49.3 cm\(^3\)
3. \((\sqrt{5338} - 37)\) cm \(\approx 36.1\) cm
4. Area = 56.57\(\pi\) cm \(\approx 177.7\) cm\(^2\)
5. \(AD = \sqrt{115.04}\) cm \(\approx 10.7\) cm
6. \(ST = 9\sqrt{3} \approx 15.6\)
7. 150\(^\circ\)

LESSON 10.1 • The Geometry of Solids
1. oblique
2. the axis
3. the altitude
4. bases
5. a radius
6. right
7. Circle C
8. A
9. AC or AC
10. BC or BC
11. Right pentagonal prism
12. ABCDE and FGHIJ
13. AF, BG, CH, DI, EJ
14. Any of AF, BG, CH, DI, EJ or their lengths
15. False. The axis is not perpendicular to the base in an oblique cylinder.
16. False. A rectanglar prism has six faces. Four are called lateral faces and two are called bases.
17. True
18. 

LESSON 10.2 • Volume of Prisms and Cylinders
1. 232.16 cm\(^3\)
2. 144 cm\(^3\)
3. 415.69 cm\(^3\)
4. \(V = 4xy(2x + 3)\), or \(8x^2y + 12xy\)
5. \(V = \frac{1}{4}p^2h\pi\)
6. \(V = \left(6 + \frac{1}{2}\pi\right)x^2y\)
7. 6 ft\(^3\)

LESSON 10.3 • Volume of Pyramids and Cones
1. 80 cm\(^3\)
2. 209.14 cm\(^3\)
3. 615.75 cm\(^3\)
4. \(V = 840x^3\)
5. \(V = \frac{8}{3}\pi a^2b\)
6. \(V = 4\pi xy^2\)
7. A: 128\(\pi\) cubic units, B: 144\(\pi\) cubic units. B is larger.
8. A: 5 cubic units, B: 5 cubic units.
9. A: 9\(\pi\) cubic units, B: 27\(\pi\) cubic units. B is larger.

LESSON 10.4 • Volume Problems
1. 4.4 cm
2. 1728 in\(^3\)
3. 24 cans; 3582 in\(^3\) = 2.07 ft\(^3\); 34.6%.
4. 2000.6 lb (about 1 ton)
5. Note that \(AE \perp AB\) and \(EC \perp BC\). \(V = \frac{8}{3}\) cm\(^3\); \(SA = \left(8 + 4\sqrt{2}\right)\) cm\(^2\) \(\approx 13.7\) cm\(^2\)
6. About 110,447 gallons
7. 57 truckloads

LESSON 10.5 • Displacement and Density
All answers are approximate.
1. 53.0 cm\(^3\)
2. 7.83 g/cm\(^3\)
3. 0.54 g/cm\(^3\)
4. 4.94 in.
5. No, it’s not gold (or at least not pure gold). The mass of the nugget is 165 g, and the volume is 17.67 cm\(^3\), so the density is 9.34 g/cm\(^3\). Pure gold has density 19.3 g/cm\(^3\).

LESSON 10.6 • Volume of a Sphere
1. 288\(\pi\) cm\(^3\), or about 904.8 cm\(^3\)
2. 18\(\pi\) cm\(^3\), or about 56.5 cm\(^3\)
3. 72\(\pi\) cm\(^3\), or about 226.2 cm\(^3\)
4. \(\frac{28}{3}\) cm\(^3\), or about 29.3 cm\(^3\)
5. 432\(\pi\) cm\(^3\), or about 1357.2 cm\(^3\)
6. \(\frac{304}{3}\) cm\(^3\), or about 318.3 cm\(^3\)
7. 11 cm
8. 2250\(\pi\) in\(^3\) \(\approx 7068.6\) in\(^3\)
9. 823.2 in\(^3\); 47.6%
10. 17.86

LESSON 10.7 • Surface Area of a Sphere
1. \(V = 1563.5\) cm\(^3\); \(S = 651.4\) cm\(^2\)
2. \(V = 184.3\) cm\(^3\); \(S = 163.4\) cm\(^2\)
3. \( V = 890.1 \text{ cm}^3; S = 486.9 \text{ cm}^2 \)
4. \( V = 34.1 \text{ cm}^3; S = 61.1 \text{ cm}^2 \)
5. About 3.9 cm
6. About 357.3 cm²
7. 9 quarts

**LESSON 11.1 • Similar Polygons**

1. \( AP = 8 \text{ cm}; EI = 7 \text{ cm}; SN = 15 \text{ cm}; YR = 12 \text{ cm} \)
2. \( SL = 5.2 \text{ cm}; MI = 10 \text{ cm}; m\measuredangle D = 120^\circ; m\measuredangle U = 85^\circ; m\measuredangle A = 80^\circ \)
3. Yes. All corresponding angles are congruent. Both figures are parallelograms, so opposite sides within each parallelogram are equal. The corresponding sides are proportional \( \left( \frac{3}{5} = \frac{4}{7} \right) \).
4. Yes. Corresponding angles are congruent by the CA Conjecture. Corresponding sides are proportional \( \left( \frac{2}{3} = \frac{3}{5} = \frac{4}{7} \right) \).
5. No. \( \frac{6}{18} \neq \frac{8}{22} \).
6. Yes. All angles are right angles, so corresponding angles are congruent. The corresponding side lengths have the ratio \( \frac{3}{5} \), so corresponding side lengths are proportional.
7. \( \frac{1}{2} \)

**LESSON 11.2 • Similar Triangles**

1. \( MC = 10.5 \text{ cm} \)
2. \( \measuredangle Q \equiv \measuredangle X; QR = 4.8 \text{ cm}; QS = 11.2 \text{ cm} \)
3. \( \measuredangle A \equiv \measuredangle E; CD = 13.5 \text{ cm}; AB = 10 \text{ cm} \)
4. \( TS = 15 \text{ cm}; QP = 51 \text{ cm} \)
5. AA Similarity Conjecture
6. \( CA = 64 \text{ cm} \)
7. \( \triangle ABC \sim \triangle EDC \). Possible explanation: \( \measuredangle A \equiv \measuredangle E \) and \( \measuredangle B \equiv \measuredangle D \) by AIA, so by the AA Similarity Conjecture, the triangles are similar.
8. \( \triangle PQR \sim \triangle STR \). Possible explanation: \( \measuredangle P \equiv \measuredangle S \) and \( \measuredangle Q \equiv \measuredangle T \) because each pair is inscribed in the same arc, so by the AA Similarity Conjecture, the triangles are similar.
9. \( \triangle MLK \sim \triangle NOK \). Possible explanation: \( \measuredangle MLK \equiv \measuredangle NOK \) by CA and \( \measuredangle K \equiv \measuredangle K \) because they are the same angle, so by the AA Similarity Conjecture, the two triangles are similar.

**LESSON 11.3 • Indirect Measurement with Similar Triangles**

1. 27 ft
2. 6510 ft
3. 110.2 mi
4. About 18.5 ft
5. 0.6 m, 1.2 m, 1.8 m, 2.4 m, and 3.0 m

**LESSON 11.4 • Corresponding Parts of Similar Triangles**

1. \( h = 0.9 \text{ cm}; j = 4.0 \text{ cm} \)
2. 3.75 cm, 4.50 cm, 5.60 cm
3. \( WX = 13\frac{5}{7} \approx 13.7 \text{ cm}; AD = 21 \text{ cm}; DB = 12 \text{ cm}; YZ = 8 \text{ cm}; XZ = 6\frac{6}{7} \approx 6.9 \text{ cm} \)
4. \( x = \frac{50}{13} \approx 3.85 \text{ cm}; y = \frac{80}{13} \approx 6.15 \text{ cm} \)
5. \( a = 8 \text{ cm}; b = 3.2 \text{ cm}; c = 2.8 \text{ cm} \)
6. \( CB = 24 \text{ cm}; CD = 5.25 \text{ cm}; AD = 8.75 \text{ cm} \)

**LESSON 11.5 • Proportions with Area**

1. \( 5.4 \text{ cm}^2 \)
2. 4 cm
3. \( \frac{9}{25} \)
4. \( \frac{36}{1} \)
5. \( \frac{25}{4} \)
6. 16:25
7. 2:3
8. 888\frac{8}{9} \text{ cm}^2 \)
9. 1296 tiles

**LESSON 11.6 • Proportions with Volume**

1. Yes
2. No
3. 16 cm³
4. 20 cm
5. \( 8:125 \)
6. 6 ft²

**LESSON 11.7 • Proportional Segments Between Parallel Lines**

1. \( x = 12 \text{ cm} \)
2. Yes
3. No
4. \( NE = 31.25 \text{ cm} \)
5. \( PR = 6 \text{ cm}; PQ = 4 \text{ cm}; RI = 12 \text{ cm} \)
6. \( a = 9 \text{ cm}; b = 18 \text{ cm} \)
7. $RS = 22.5\,\text{cm}, \, EB = 20\,\text{cm}$
8. $x = 20\,\text{cm}; \, y = 7.2\,\text{cm}$
9. $p = \frac{16}{3} = 5.3\,\text{cm}; \, q = \frac{8}{3} = 2.6\,\text{cm}$

LESSON 12.1 • Trigonometric Ratios
1. $\sin P = \frac{P}{r}$
2. $\cos P = \frac{q}{r}$
3. $\tan P = \frac{p}{q}$
4. $\sin Q = \frac{q}{r}$
5. $\sin T = 0.800$
6. $\cos T = 0.600$
7. $\tan T \approx 1.333$
8. $\sin R = 0.600$
9. $x \approx 12.27$
10. $x \approx 29.75$
11. $x \approx 18.28$
12. $\angle A \approx 71^\circ$
13. $\angle B \approx 53^\circ$
14. $\angle C \approx 30^\circ$
15. $\sin 40^\circ = \frac{w}{28}; \, w \approx 18.0\,\text{cm}$
16. $\sin 28^\circ = \frac{x}{14}; \, x \approx 7.4\,\text{cm}$
17. $\cos 17^\circ = \frac{y}{7}; \, y \approx 76.3\,\text{cm}$
18. $a \approx 28^\circ$
19. $t \approx 47^\circ$
20. $z \approx 76^\circ$

LESSON 12.2 • Problem Solving with Right Triangles
1. Area $\approx 2\,\text{cm}^2$
2. Area $\approx 325\,\text{ft}^2$
3. Area $\approx 109\,\text{in}^2$
4. $x \approx 54.0^\circ$
5. $y \approx 31.3^\circ$
6. $a \approx 7.6\,\text{in.}$
7. Diameter $\approx 20.5\,\text{cm}$
8. $\theta \approx 45.2^\circ$
9. $\beta \approx 28.3^\circ$
10. About 2.0 m
11. About 445.2 ft
12. About 22.6 ft

LESSON 12.3 • The Law of Sines
1. Area $\approx 46\,\text{cm}^2$
2. Area $\approx 24\,\text{m}^2$
3. Area $\approx 45\,\text{ft}^2$
4. $m \approx 14\,\text{cm}$
5. $p \approx 17\,\text{cm}$
6. $q \approx 13\,\text{cm}$
7. $\angle B \approx 66^\circ, \, \angle C \approx 33^\circ$
8. $\angle P \approx 37^\circ, \, \angle Q \approx 95^\circ$
9. $\angle K \approx 81^\circ, \, \angle M \approx 21^\circ$
10. Second line: about 153 ft, between tethers: about 135 ft

LESSON 12.4 • The Law of Cosines
1. $t \approx 13\,\text{cm}$
2. $b \approx 67\,\text{cm}$
3. $w \approx 34\,\text{cm}$
4. $\angle A \approx 76^\circ, \, \angle B \approx 45^\circ, \, \angle C \approx 59^\circ$
5. $\angle A \approx 77^\circ, \, \angle P \approx 66^\circ, \, \angle S \approx 37^\circ$
6. $\angle S \approx 46^\circ, \, \angle U \approx 85^\circ, \, \angle V \approx 49^\circ$

LESSON 12.5 • Problem Solving with Trigonometry
1. About 2.85 mi/h; about 15°
2. $\angle A \approx 50.64^\circ, \, \angle B \approx 59.70^\circ, \, \angle C \approx 69.66^\circ$
3. About 8.0 km from Tower 1, 5.1 km from Tower 2
4. About 853 miles
5. About 530 ft of fencing; about 11,656 ft²

LESSON 13.1 • The Premises of Geometry
1. a. Given
   b. Distributive property
   c. Subtraction property
   d. Addition property
   e. Division property
2. False

4. True; transitive property of congruence and definition of congruence

5.

LESSON 13.2 • Planning a Geometry Proof

Proofs may vary.

1. Flowchart Proof
2. Proof:

Statement | Reason
--- | ---
1. \( CD \parallel BD \) | 1. Given
2. \( BD \perp AB \) | 2. Given
3. \( CD \perp AC \) | 3. Given
4. \( AD \) is bisector of \( \angle CAB \) | 4. Converse of Angle Bisector Theorem
5. \( \angle CAD \equiv \angle BAD \) | 5. Definition of angle bisector
6. \( \angle ACD \) is a right angle | 6. Definition of perpendicular
7. \( \angle ABD \) is a right angle | 7. Definition of perpendicular
8. \( \angle ACD \equiv \angle ABD \) | 8. Right Angles Are Congruent Theorem
9. \( \triangle ABD \equiv \triangle ACD \) | 9. SAA Theorem

3. Flowchart Proof

Given \( \angle PQR \equiv \angle TSU \)

AIA Theorem

\( \angle QRP \equiv \angle TUS \)

Third Angle Theorem

Converse of AIA Theorem

3. Flowchart Proof

Given \( \triangle ABC \)

VA Theorem

\( a = x \)

\( x + b + c = 180^\circ \)

Triangle Sum Theorem

Substitution

\( b = y \)

VA Theorem

Substitution

\( c = z \)

VA Theorem

LESSON 13.3 • Triangle Proofs

Proofs may vary.

1. Flowchart Proof

Given \( XY = YZ \)

Definition of isosceles triangle

\( \triangle XZY \) is isosceles

Definition of altitude from vertex \( Y \)

\( TM \) is the altitude from vertex \( Y \)

Definition of altitude and vertex angle

\( TM \) is angle bisector of \( \angle XYZ \)

Isosceles Triangle

\( \triangle XZY \) isosceles

Vertex Angle Theorem

\( \angle XYM \equiv \angle ZYM \)

Definition of angle bisector

4. Proof:

Statement | Reason
--- | ---
1. \( AB \equiv BC \) | 1. Given
2. \( \triangle ABC \) is isosceles | 2. Definition of isosceles triangle
3. \( \angle A \equiv \angle ACB \) | 3. IT Theorem
4. \( \angle ACB \equiv \angle DCE \) | 4. Given
5. \( \angle A \equiv \angle DCE \)
6. \( \overline{AB} \parallel \overline{CE} \)
7. \( \angle ABD \equiv \angle CED \)
8. \( \overline{AB} \perp \overline{BD} \)
9. \( \angle ABD \) is a right angle
10. \( \angle CED \) is a right angle
11. \( \overline{BD} \perp \overline{CE} \)

**LESSON 13.4 - Quadrilateral Proofs**

Proofs may vary.

1. **Given:** \( ABCD \) is a parallelogram  
   **Show:** \( AC \) and \( BD \) bisect each other at \( M \)  
   **Flowchart Proof**

2. **Given:** \( DM = BM, AM = CM \)  
   **Show:** \( ABCD \) is a parallelogram  
   **Proof:**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( DM = BM )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( DM \equiv BM )</td>
<td>2. Definition of congruence</td>
</tr>
</tbody>
</table>
Lesson 13.4, Exercise 4

4. Given: \( \overline{AC} \) and \( \overline{BD} \) bisect each other at \( M \) and \( \overline{AC} \perp \overline{BD} \)

\[ \text{Show: } ABCD \text{ is a rhombus} \]

Flowchart Proof

(See flowchart at bottom of page.)

5. Given: \( ABCD \) is a trapezoid with \( \overline{AB} \parallel \overline{CD} \) and \( \angle A \equiv \angle B \)

\[ \text{Show: } ABCD \text{ is isosceles} \]

Proof:

\[ \begin{array}{c|c}
\text{Statement} & \text{Reason} \\
1. ABCD \text{ is a trapezoid} & 1. \text{Given} \\
2. \overline{CE} \parallel \overline{AD} & 2. \text{Parallel Postulate} \\
3. \triangle ACE \text{ is a parallelogram} & 3. \text{Definition of parallelogram} \\
4. \overline{AD} \equiv \overline{CE} & 4. \text{Opposite Sides Congruent Theorem} \\
5. \angle A \equiv \angle BEC & 5. \text{CA Postulate} \\
6. \angle A \equiv \angle B & 6. \text{Given} \\
7. \angle BEC \equiv \angle B & 7. \text{Transitivity} \\
8. \triangle EBC \text{ is isosceles} & 8. \text{Converse of IT Theorem} \\
9. \overline{EC} \equiv \overline{CB} & 9. \text{Definition of isosceles triangle} \\
10. \overline{AD} \equiv \overline{CB} & 10. \text{Transitivity} \\
11. ABCD \text{ is isosceles} & 11. \text{Definition of isosceles trapezoid} \\
\end{array} \]

6. Given: \( ABCD \) is a trapezoid with \( \overline{AB} \parallel \overline{CD} \) and \( \overline{AC} \equiv \overline{BD} \)

\[ \text{Show: } ABCD \text{ is isosceles} \]

Proof:

\[ \begin{array}{c|c}
\text{Statement} & \text{Reason} \\
1. ABCD \text{ is a trapezoid} & 1. \text{Given} \\
2. \overline{BE} \parallel \overline{AC} & 2. \text{Parallel Postulate} \\
3. \overline{DC} \text{ and } \overline{BE} \text{ intersect at } F & 3. \text{Line Intersection Postulate} \\
4. \triangle ABF \text{ is a parallelogram} & 4. \text{Definition of parallelogram} \\
5. \overline{AC} \equiv \overline{BF} & 5. \text{Opposite Sides Congruent Theorem} \\
6. \overline{AC} \equiv \overline{BD} & 6. \text{Given} \\
7. \overline{BF} \equiv \overline{BD} & 7. \text{Transitivity} \\
8. \triangle DFB \text{ is isosceles} & 8. \text{Definition of isosceles triangle} \\
9. \angle DFB \equiv \angle FDB & 9. \text{IT Theorem} \\
10. \angle CAB \equiv \angle DFB & 10. \text{Opposite Angles Theorem} \\
11. \angle DFB \equiv \angle DBA & 11. \text{AIA Theorem} \\
12. \angle CAB \equiv \angle DBA & 12. \text{Transitivity} \\
13. \overline{AB} \equiv \overline{AB} & 13. \text{Reflexive property} \\
14. \triangle ACB \equiv \triangle BDA & 14. \text{SAS Postulate} \\
15. \overline{AD} \equiv \overline{BC} & 15. \text{CPCTC} \\
16. ABCD \text{ is isosceles} & 16. \text{Definition of isosceles trapezoid} \\
\end{array} \]
7. False

8. False

9. True

**Given:** ABCD with \( AB \parallel CD \) and \( \angle A \equiv \angle C \)

**Show:** ABCD is a parallelogram

**Flowchart Proof**

- **Given:** \( AB \parallel CD \)
- **Interior Supplements Theorem:** \( \angle A \) and \( \angle D \) are supplementary
- **Interior Supplements Theorem:** \( \angle C \) and \( \angle B \) are supplementary
- **Given:** \( \angle A \equiv \angle C \)
- **Supplements of Congruent Angles Theorem:** \( \angle D \equiv \angle B \)
- **Converse of Opposite Angles Theorem:** ABCD is a parallelogram

Therefore the assumption, \( BC \leq AC \), is false, so \( BC > AC \).

**2. Paragraph Proof:** Assume \( \angle DAC \equiv \angle BAC \)

It is given that \( \overline{AD} \equiv \overline{AB} \). By the reflexive property \( \overline{AC} \equiv \overline{AC} \). So by SAS, \( \triangle ADC \equiv \triangle ABC \). Then \( DC \equiv BC \) by CPCTC. But this contradicts the given that \( DC \neq BC \). So \( \angle DAC \neq \angle BAC \).

**3. Given:** \( \triangle ABC \) with \( \overline{AB} \equiv \overline{BC} \)

**Show:** \( \angle C \neq \angle A \)

**Paragraph Proof:** Assume \( \angle C \equiv \angle A \)

If \( \angle C \equiv \angle A \), then by the Converse of the IT Theorem, \( \triangle ABC \) is isosceles and \( \overline{AB} \equiv \overline{BC} \). But this contradicts the given that \( \overline{AB} \equiv \overline{BC} \). Therefore, \( \angle C \neq \angle A \).

**4. Given:** Coplanar lines \( k \), \( \ell \), and \( m \), \( k \parallel \ell \), and \( m \)

**Show:** \( m \) intersects \( \ell \)

**Paragraph Proof:** Assume \( m \) does not intersect \( \ell \)

If \( m \) does not intersect \( \ell \), then by the definition of parallel, \( m \parallel \ell \). But because \( k \parallel \ell \), by the Parallel Transitivity Theorem, \( k \parallel m \). This contradicts the given that \( m \) intersects \( k \). Therefore, \( m \) intersects \( \ell \).

**LESSON 13.6 • Circle Proofs**

**1. Given:** Circle \( O \) with \( \overline{AB} \equiv \overline{CD} \)

**Show:** \( \overline{AB} \equiv \overline{CD} \)

**Flowchart Proof**

- **Construct:** \( OA, OB, OC, OD \)
- **Line Postulate:** \( OA = OD \)
- **Definition of circle, definition of radii:** \( \overline{AB} \equiv \overline{CD} \)
- **Definition of circle, definition of radii:** \( \overline{OB} = \overline{OC} \)
- **SSS Postulate:** \( \triangle OAB \equiv \triangle ODC \)
- **CPCTC:** \( \angle AOB \equiv \angle DOC \)
- **Definition of congruence, definition of arc measure, transitivity:** \( AB = CD \)
2. **Paragraph Proof:** Chords $BC$, $CD$, and $DE$ are congruent because the pentagon is regular. By the proof in Exercise 1, the arcs $BC$, $CD$, and $DE$ are congruent and therefore have the same measure. $m \angle EAD = \frac{1}{2} m \angle DBE$ by the Inscribed Angles Intercepting Arcs Theorem. Similarly, $m \angle DAC = \frac{1}{2} m \angle DCB$ and $m \angle BAC = \frac{1}{2} m \angle BCD$. By transitivity and algebra, the three angles have the same measure. So, by the definition of trisect, the diagonals trisect $\angle BAE$.

3. **Paragraph Proof:** Construct the common internal tangent $\overline{RU}$ (Line Postulate, definition of tangent). Label the intersection of the tangent and $\overline{TS}$ as $U$.

\[ \overline{TU} \equiv \overline{RU} \equiv \overline{SU} \] by the Tangent Segments Theorem. $\triangle TUR$ is isosceles by definition because $\overline{TU} \equiv \overline{RU}$. So, by the IT Theorem, $\angle T \equiv \angle TRU$. Call this angle measure $x$. $\triangle SUR$ is isosceles because $\overline{RU} \equiv \overline{SU}$, and by the IT Theorem, $\angle S \equiv \angle URS$. Call this angle measure $y$. The angle measures of $\triangle TRS$ are then $x$, $y$, and $(x + y)$. By the Triangle Sum Theorem, $x + y + (x + y) = 180^\circ$. By algebra (combining like terms and dividing by 2), $x + y = 90^\circ$. But $m \angle TRS = x + y$, so by transitivity and the definition of right angle, $\angle TRS$ is a right angle.

4. **Paragraph Proof:** Construct tangent $\overline{TP}$ (Line Postulate, definition of tangent). $\angle PTD$ and $\angle TAC$ both have the same intercepted arc, $\overline{TC}$. Similarly, $\angle PTD$ and $\angle TBD$ have the same intercepted arc, $\overline{TD}$. So, by transitivity, the Inscribed Angles Intercepting Arcs Theorem, and algebra, $\angle TAC$ and $\angle TBD$ are congruent. Therefore, by the Converse of the CA Postulate, $\overline{AC} \parallel \overline{BD}$.
3. Given: $\triangle ABC$ with $\angle ACB$ right, $CD \perp AB$

Show: $AC \cdot BC = AB \cdot CD$

Flowchart Proof

```
CD \perp AB
  Given
  \angle ADC is right
  Definition of perpendicular
  \angle ADC = \angle ACB
    Right Angles Are Congruent Theorem
  \triangle ACB \cong \triangle ADC
    AA Similarity Postulate
  \frac{AC}{AB} = \frac{CD}{BC}
    Definition of similarity
  AC \cdot BC = AB \cdot CD
    Multiplication property
```

4. Given: $ABCD$ with right angles $A$ and $C$, $\overline{AB} \cong \overline{DC}$

Show: $ABCD$ is a rectangle

Proof:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle ABC$ with $\angle ACB$ right, $CD \perp AB$</td>
<td>$\angle A$ and $\angle C$ are right angles</td>
</tr>
<tr>
<td>$\angle A \cong \angle C$</td>
<td>Right Angles Are Congruent Theorem</td>
</tr>
<tr>
<td>$\overline{AB} \cong \overline{DC}$</td>
<td>Given</td>
</tr>
<tr>
<td>$\overline{DB} \cong \overline{DB}$</td>
<td>Reflexive property</td>
</tr>
<tr>
<td>$\triangle DBA \cong \triangle BDC$</td>
<td>HL Congruence Theorem</td>
</tr>
<tr>
<td>$\angle DBA \cong \angle BDC$</td>
<td>CPCTC</td>
</tr>
<tr>
<td>$m\angle DBA + m\angle DBA + m\angle C = 180^\circ$</td>
<td>Triangle Sum Theorem</td>
</tr>
<tr>
<td>$m\angle A = 90^\circ$</td>
<td>Definition of right angle</td>
</tr>
<tr>
<td>$m\angle A + m\angle DBA = 90^\circ$</td>
<td>Subtraction property</td>
</tr>
<tr>
<td>$m\angle A + m\angle BDC = 90^\circ$</td>
<td>Substitution</td>
</tr>
<tr>
<td>$m\angle ADC = 90^\circ$</td>
<td>Angle Addition Postulate</td>
</tr>
<tr>
<td>$m\angle C = 90^\circ$</td>
<td>Transitivity</td>
</tr>
<tr>
<td>$m\angle A + m\angle ABC + m\angle C + m\angle ADC = 360^\circ$</td>
<td>Definition of right angle</td>
</tr>
<tr>
<td>$m\angle A = 90^\circ$</td>
<td>Quadrilateral Sum Theorem</td>
</tr>
<tr>
<td>$m\angle A \cong \angle ABC \cong \angle C \cong \angle ADC$</td>
<td>Substitution property and subtraction property</td>
</tr>
<tr>
<td>$ABCD$ is a rectangle</td>
<td>Definition of congruence</td>
</tr>
</tbody>
</table>

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